Exponential Functions

(Section 7.1)

I can determine if a function represents exponential growth or decay.

I can graph exponential growth and decay functions using transformations.

Exponential Functions → "x" is in exponent

growth

decay

$$f(x) = ab^{x-h} + k \qquad a \neq 0,\ b > 0,\ \text{and } b \neq 1.$$

a = 1 b = 2

Exponential growth, exponential decay, or neither:

- If a>0 and b>1, then exponential growth.
  - b is between 0 + 1
- If a>0 and 0<b<1, then exponential decay.
- If a<0, then neither growth or decay.
  - neg

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$$f(x) = ab^{x-h} + k \qquad a \neq 0,\ b > 0,\ \text{and}\ b \neq 1.$$

$$y = ab^{x-h} + k$$

never ends

x + y values do not repeat

1. Function is continuous and one-to-one.
2. Domain = $(-\infty, +\infty)$
3. y = k is the asymptote of the graph.
4. Range: $[k, +\infty)$ if a>0

   $(-\infty, k]$ if a<0

HA: Horizontal Asymptote

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$$f(x) = ab^{x-h} + k$$ **$a \neq 0$, $b > 0$, and $b \neq 1$.**

Transformations:

★ **h - translation left/right**

$x - h$: shift right h units

$x + h$: shift left h units

★ **k - translation up/down**

$+ k$: shift up k units

$- k$: shift down k units

★ **a - reflection in x-axis, vertical compression/stretch**

$a < 0$: reflection in x-axis

negative

$y = \frac{1}{2}\left(\frac{2}{3}\right)^x$

$|a| > 1$: vertical stretch

$0 < |a| < 1$: vertical compression

TRANSFORMATION GUIDE POINTS: $\left(-1, \frac{1}{b}\right)$ $(0, 1)$ $(1, b)$

Exponential Functions

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I can determine if a function represents exponential growth or decay.

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$$f(x) = ab^{x-h} + k \qquad \textbf{a} \neq 0,\ b > 0, \text{ and } b = 1.$$

Examples: Determine whether each function represents growth, decay or neither. Describe the transformation(s) from the parent graph.

1.a. $y = (0.7)^{x-4} + 3$

a = 1

b = 0.7

Decay

Transformation(s):

$y = (0.7)^x$ has been shifted right 4 and up 3

b. $y = \frac{1}{2}(3)^x$

$a = \frac{1}{2}$

b = 3

Growth

Transformation(s):

$y = 3^x$ has been vert. compres

c. $y = 10(\frac{4}{3})^x - 2$

a = 10

$b = \frac{4}{3} = 1\frac{1}{3}$

Growth

Transformation(s):

$y = \frac{4}{3}^x$ has been vert stretched by 10 and moved down 2.

d. $y = -10(\frac{4}{3})^{x+5}$

a = −10

Neither

Transformation(s):

$y = \frac{4}{3}^x$ has moved left 5 been stretched vert. by 10 and reflected in x-axis.

Exponential Functions

(Section 7.1)

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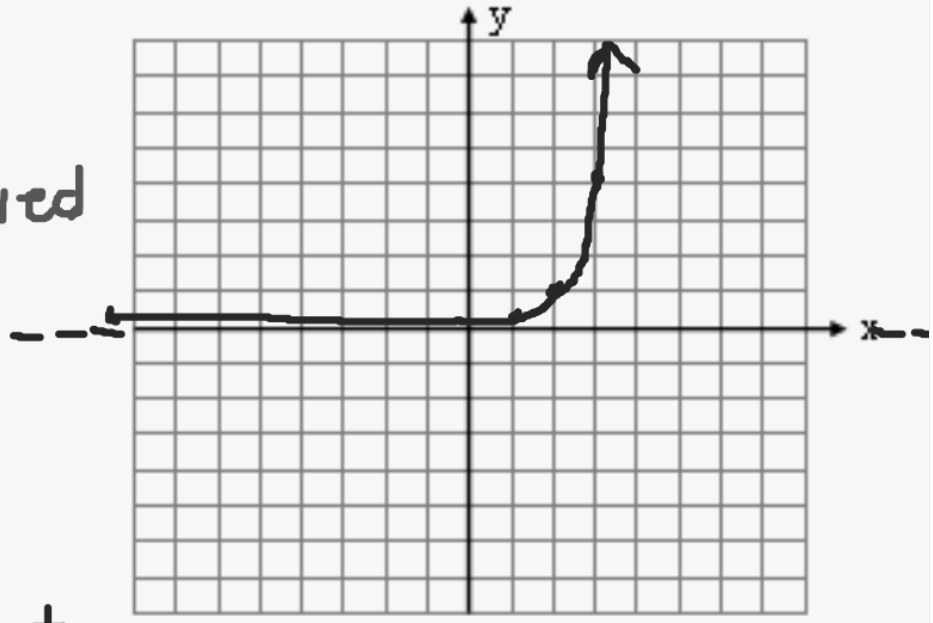
*Describe the transformation(s) and use the guide points with transformations to graph the function. State the domain and range.

2. $y = 4^{x-2}$

$a = 1$ Growth

$b = 4$

Transformation(s): $y = 4^x$ has moved right 2.



Guide Points:

$(-1, \frac{1}{b})$ $(0, 1)$ $(1, b)$

$(-1, \frac{1}{4})$ $(0, 1)$ $(1, 4)$ start

+2 +2 +2

$(1, \frac{1}{4})$ $(2, 1)$ $(3, 4)$ HA: $y = 0$

Domain: $(-\infty, +\infty)$

Range: $(0, +\infty)$

Exponential Functions

(Section 7.1)

I can determine if a function represents exponential growth or decay.

I can graph exponential growth and decay functions using transformations.

3. $y = \left(\frac{1}{2}\right)^{x-3} + 2$

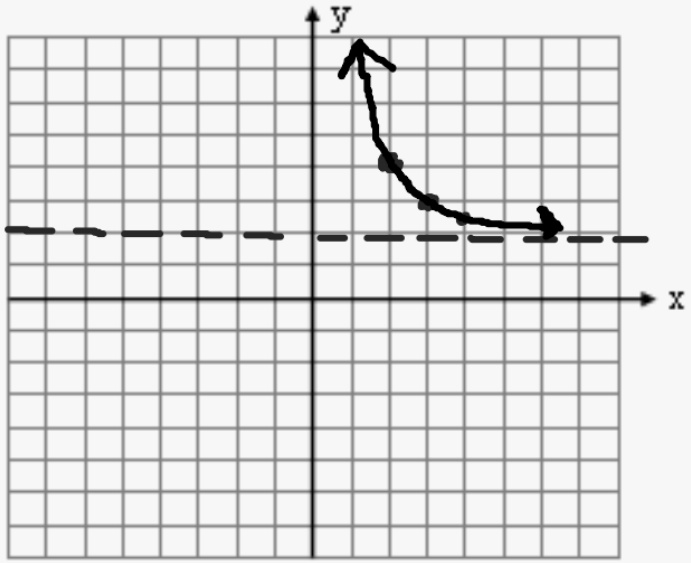
$a = 1$

$b = \frac{1}{2}$

Decay

k

Transformation(s): $y = \left(\frac{1}{2}\right)^x$ has moved 3 to right and up 2



Guide Points:

$\left(-1, \frac{1}{\frac{1}{2}}\right)$ $(0, 1)$ $\left(1, \frac{1}{2}\right)$

$(-1, 2)$ $(0, 1)$ $\left(1, \frac{1}{2}\right)$

$+3$ $+3$ $+3$

$(2, 2)$ $(3, 1)$ $\left(4, \frac{1}{2}\right)$

$+2$ $+2$ $+2$

$(2, 4)$ $(3, 3)$ $(4, 2.5)$ ✗

HA: $y = 2$

Domain: $(-\infty, +\infty)$

Range: $(2, +\infty)$

Exponential Functions

(Section 7.1)

I can determine if a function represents exponential growth or decay.

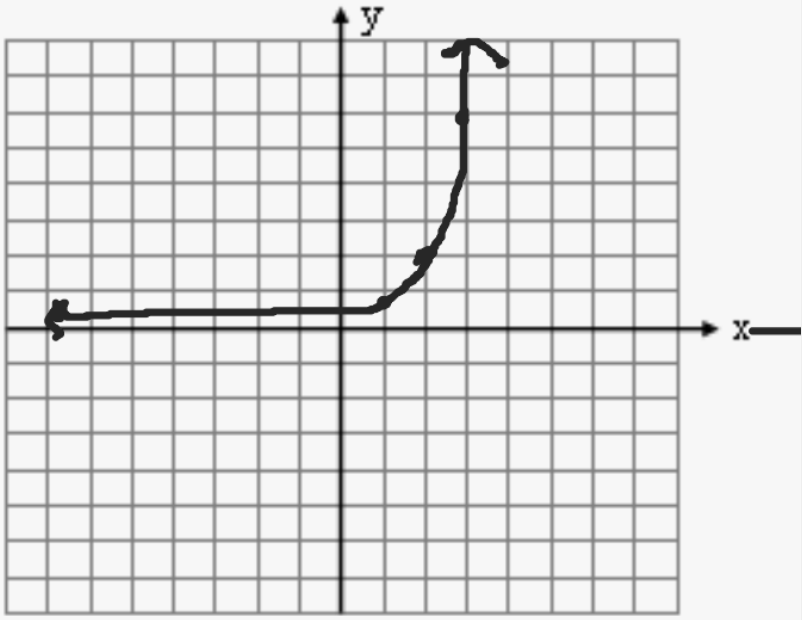
I can graph exponential growth and decay functions using transformations.

4. $y = 2(3)^{x+2}$

$a=2$ Growth

$b=3$

Transformation(s): $y=3^x$ has moved left 2 and vert. stretched by 2.



Guide Points:

$(-1, \frac{1}{3})$ $(0,1)$ $(1,3)$

$+2$ $+2$ $+2$

$(1, \frac{1}{3})$ $(2,1)$ $(3,3)$

$\times 2$ $\times 2$ $\times 2$

$(1, \frac{2}{3})$ $(2,2)$ $(3,6)$ HA: $y=0$

Domain: $(-\infty, +\infty)$

Range: $(0, +\infty)$

Exponential Functions

(Section 7.1)

I can determine if a function represents exponential growth or decay.

I can graph exponential growth and decay functions using transformations.

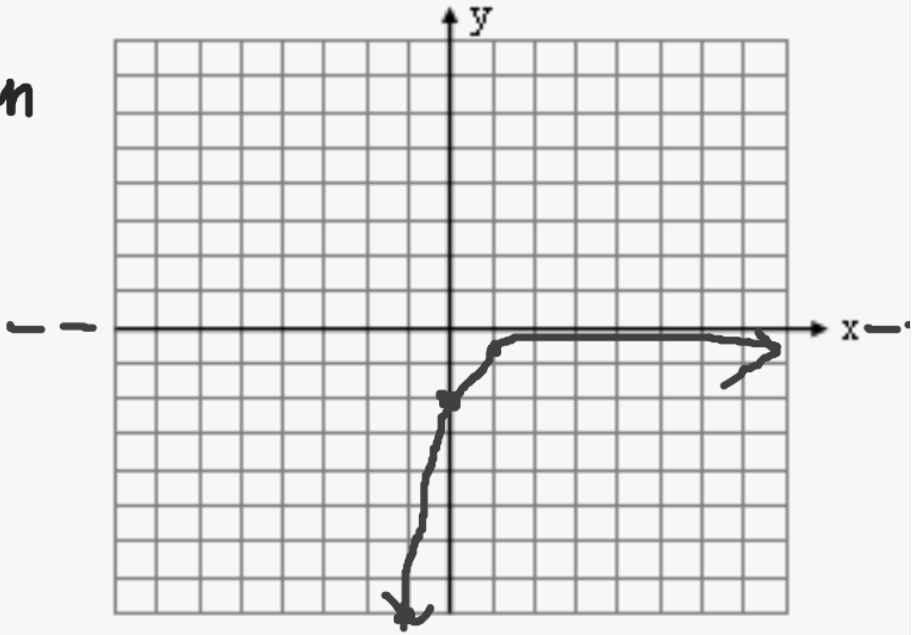
5. $y = -2\left(\frac{1}{4}\right)^x$

Neither

$a = -2$

$b = \frac{1}{4}$

Transformation(s): $y = \frac{1}{4}^x$ has been vert. stretched by 2 + reflected over x-axis,



Guide Points:

$(-1, \frac{1}{\frac{1}{4}})$ $(0, 1)$ $(1, \frac{1}{4})$

$(-1, 4)$ $(0, 1)$ $(1, \frac{1}{4})$

$\times 2$ $\times 2$ $\times 2$

$(-1, 8)$ $(0, 2)$ $(1, \frac{1}{2})$

$x-1$ $x-1$ $x-1$

$(-1, -8)$ $(0, -2)$ $(1, -\frac{1}{2})$

HA: $y = 0$

Domain: $(-\infty, +\infty)$

Range: $(-\infty, 0)$

Exponential Functions

(Section 7.1)

I can determine if a function represents exponential growth or decay.

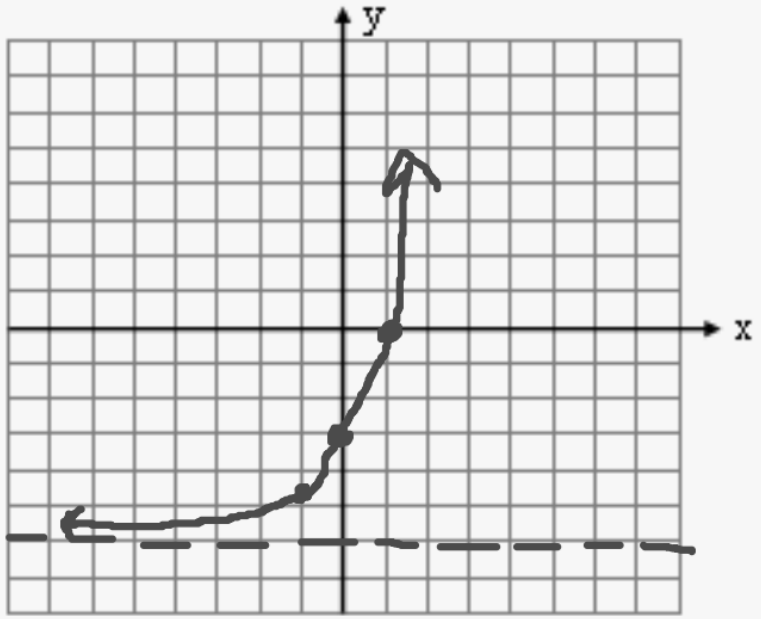
I can graph exponential growth and decay functions using transformations.

6. $y = 3(2)^x - 6$

$a=3$ Growth

$b=2$

Transformation(s): $y=2^x$ has been vert stretch by 3 and move down 6.



Guide Points:

$(-1, \frac{1}{2})$ $(0,1)$ $(1,2)$

$\times 3$ $\times 3$ $\times 3$

$(-1, \frac{3}{2})$ $(0,3)$ $(1,6)$

-6 -6 -6

$(-1, -\frac{9}{2})$ $(0,-3)$ $(1,0)$

$(-1, -4\frac{1}{2})$

HA: $y=-6$

Domain: $(-\infty, +\infty)$

Range: $(-6, +\infty)$

Exponential Functions

(Section 7.1)

I can determine if a function represents exponential growth or decay.

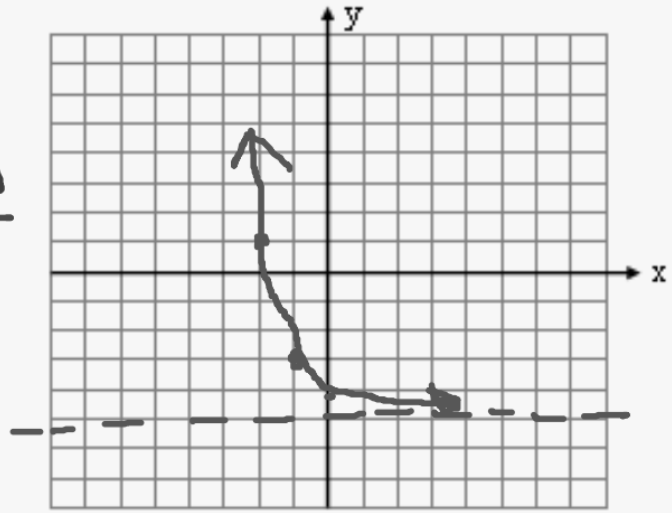
I can graph exponential growth and decay functions using transformations.

7. $y = 2\left(\frac{1}{3}\right)^{x+1} - 5$

$a = 2$

$b = \frac{1}{3}$ Decay

Transformation(s): $y = \frac{1}{3}^x$ has moved left one, been vert. stretched by 2, and moved down 5.



Guide Points:

$\left(-1, \frac{1}{\frac{1}{3}}\right)$ $(0, 1)$ $\left(1, \frac{1}{3}\right)$

$(-1, 3)$ $(0, 1)$ $\left(1, \frac{1}{3}\right)$

-1 -1 -1

$(-2, 3)$ $(-1, 1)$ $(0, \frac{1}{3})$

$\times 2$ $\times 2$ $\times 2$

$(-2, 6)$ $(-1, 2)$ $(0, \frac{2}{3})$

-5 -5 -5

$(-2, 1)$ $(-1, -3)$ $\left(0, -\frac{13}{3}\right) \rightarrow \left(0, -4\frac{1}{3}\right)$

#A: $y = -5$

Domain: $(-\infty, +\infty)$

Range: $(-5, +\infty)$