Unit 8 Lesson 1: Counting Techniques

Learning Targets: I can define the counting principle.
I can compare and contrast permutations and combinations.
I can use the counting principle, permutations, and combinations to determine possible outcomes.

Example 1: How many ways can you arrange the letters A, B and C?

ABC BAC CAB = 6 ways

ACB BCA CBA

$3 \cdot 2 \cdot 1 = 6$

<u>The fundamental counting principle</u> calculates the number of possible outcomes by <u>multiplying</u> the <u>possible outcomes</u> for <u>each</u> e<u>vent/decisi</u>on

Use when picking one object from different groups.

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Example 2: Jason is ordering a sandwich from a deli. There are 9 meat options, 2 types of bread and 5 different cheeses. How many different sandwiches can Jason order, if he orders only one of each choice?

meat bread cheese

$9 \cdot 2 \cdot 5 = \boxed{90}$

Example 3: Shelby is dressing a mannequin for a store window. She can choose from 2 different t-shirts and 3 styles of jeans. Each mannequin also needs a jacket, of which there are 5 styles available. How many ways can Shelby dress the mannequin?

$2 \cdot 3 \cdot 5 = \boxed{30}$

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Example 4: If there are 8 finalists in a band competition, how many different ways can the bands be ranked?

$8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = \boxed{40{,}320}$

- **What makes the two groups different?**

Permutation

How many different ways can two students be assigned to five tutors if only one student is assigned to each tutor?

T... eir n... en t... c... c... st prize, a new graphing calculator; 2nd prize, a "no homework" coupon; 3rd prize, a new pencil.

There are 5 marching bands in a parade and all of them want to be at the beginning of the parade. How many ways are there to order the 5 bands for the parade?

Combination

A student gets to select 2 items from a prize bucket that contains 6 different prizes. How many different pairs of prizes can she select?

How many ways are there to choose 5 cards from a standard deck of 52 cards?

... select a committee of **3** people. How many ways can they select the committee?

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Factorial #! : the product of an integer and all integers below it

The product 8·7·6·5·4·3·2·1 can also be written 8!
("eight factorial")

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Permuation: an arrangement of a distinct group of objects in which the **ORDER MATTERS!**

☆ The **number of permutations** of n distinct objects is **n!**

KeyConcept Permutations of *n* Objects Taken *r* at a Time

The number of permutations of *r* objects taken from a group of *n* distinct objects is given by

$$_nP_r = \frac{n!}{(n-r)!}.$$

total # in group; # we select

$$_8P_4 = \frac{8!}{(8-4)!} = \frac{8!}{4!} = 1680$$

$$\frac{8!}{4!} = \frac{8\cdot7\cdot6\cdot5\cdot4\cdot3\cdot2}{4\cdot3\cdot2\cdot1}$$

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Combination: **a selection of distinct objects in which the ORDER does NOT matter!**

KeyConcept **Combinations of n Objects Taken r at a Time**

The number of combinations of r objects taken from a group of n distinct objects is given by

$$_nC_r = \frac{_nP_r}{r!} = \frac{n!}{(n-r)!\,r!}$$

$$\frac{8!}{2!\,6!} \qquad \frac{8\cdot 7\cdot 6\cdot 5\cdot 4\cdot 3\cdot 2}{2\cdot 6\cdot 5\cdot 4\cdot 3\cdot 2} = 28$$

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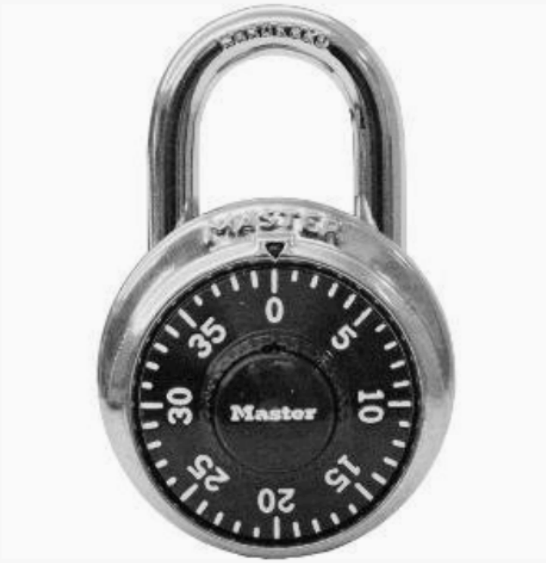

What is this?

combination lock

$${}_{40}P_3 = 59{,}280$$

$$\frac{40!}{37!}$$

WE DO!

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Determine whether each situation involves permutations or combinations. Then solve the problem.

Example 5: A company with 45 employees wants to select a committee of **3** people. How many ways can they select the committee?

no order → Combination

$${}_{45}C_3 = \frac{45!}{42!\,3!} = \boxed{14,190}$$

Example 6: A company with 45 employees wants to select a committee with **3** positions: a president, treasurer and secrtary. How many options are there?

order – permutations

$${}_{45}P_3 = \frac{45!}{42!} = \boxed{85,140}$$

WE DO!

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Example 7: Twenty-five students write their names on slips of paper. Then three different names are chosen at random to receive a "no homework" coupon.

no order - comb,

$$_{25}C_3 = \frac{25!}{22!3!} = \boxed{2{,}300}$$

Example 8: Twenty-five students write their names on slips of paper. Then three different names are chosen at random to receive one of the following prizes: 1st prize, a new graphing calculator; 2nd prize, a "no homework" coupon; 3rd prize, a new pencil. order – Perm

$$_{25}P_3 = \frac{25!}{22!} = 13{,}800$$

WE DO!

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Example 9: How many different ways can two students be assigned to five tutors if only one student is assigned to each tutor?

Order → Perm

$$_5P_2 = \frac{5!}{3!} = \boxed{20}$$

Example 10: How many ways are there to choose 5 cards from a standard deck of 52 cards?

no order – comb

$$_{52}C_5 = \frac{52!}{47!\,5!} = \boxed{2{,}598{,}960}$$

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YOU DO!

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Example 11: There are 15 kids and you need to pick 9 to play a game of softball and assign positions. How many ways can you pick?

permutation

$${}_{15}P_{9} = \frac{15!}{6!} = 1,816,214,400$$

Example 12: There are 15 kids and you need to pick 8 to play a game of dodgeball. How many ways can you pick?

combination

$${}_{15}C_{8} = \frac{15!}{7!8!} = 6435$$

YOU DO!

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Example 13: There are 8 finalists in a band competition. How many ways can the first, second, and third place rankings be awarded?

perm.

$$_8P_3 = \frac{8!}{5!} = 336$$