Remainder and Factor Theorems

Obj: Use synthetic division/substitution to verify factors/zeros of a polynnomial.

Obj: Use synthetic/long division to depress polynomials so they can be factored completely.

Obj: Find the solutions to an equation given some or all the factors.

Remainder Theorem

If polynomial $f(x)$ is divided by $x - a$, the remainder is the constant $f(a)$.

....so when dividing the $f(x)$ by $x - 3$, the remainder is the same as finding $f(3)$.

Synthetic Substitution ⇨ applying Remainder Thm

Use synthetic substitution to evaluate functions.

1. If $f(x) = 2x^3 - 2x^2 + 3x - 7$, find $f(3)$.

$$\begin{array}{r|rrrr} 3 & 2 & -2 & 3 & -7 \\ & & +6 & 12 & 45 \\ \hline & 2 & 4 & 15 & 38 \end{array}$$

$f(3) = 38$

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Factor Theorem

The binomial $x - a$ is a factor of the polynomial $f(x)$ if and only if $f(a)=0$.

...so x - 3 is a factor of $f(x)$ if $f(3)=0$.

Use synthetic *division to depress polynomials, then factor completely.*

2. Show that x - 3 is a factor of $x^3 + 4x^2 - 15x - 18$. Then find the remaining factors of the polynomial.

$x+5$

5

3 | 1 4 -15 -18

+3 +21 +18

1 7 6 | 0

(x-3) is a factor

x^2+7x+6

$(x+6)(x+1)$

$(x+6)(x+1)(x-3)$

Concept Check: Remainder and Factor Theorems

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YOUR TURN: Use the Remainder Therorem (synthetic substitution).

1. If $f(x) = 3x^3 - 6x^2 + x - 11$, find $f(3)$.

$$\begin{array}{r|rrrr} 3 & 3 & -6 & 1 & -11 \\ & & 9 & 9 & 30 \\ \hline & 3 & 3 & 10 & 19 \end{array}$$

$$f(3) = 19$$

2. If $f(x) = 3x^5 + 2x^3 + x^2 - 1$, find $f(-1)$.

$$\begin{array}{r|rrrrrr} -1 & 3 & 0 & 2 & 1 & 0 & -1 \\ & & -3 & 3 & -5 & 4 & -4 \\ \hline & 3 & -3 & 5 & -4 & 4 & -5 \end{array}$$

$$f(-1) = -5$$

3. Show that $x - 2$ is a factor of $x^3 - 7x^2 + 4x + 12$. Then find the remaining factors of the polynomial.

$$\begin{array}{r|rrrr} 2 & 1 & -7 & 4 & 12 \\ & & 2 & -10 & -12 \\ \hline & 1 & -5 & -6 & 0 \end{array}$$

$f(2) = 0 \ \therefore\ (x-2)$ **is a factor**

$$x^2 - 5x - 6$$

$$\boxed{(x-6)(x+1)(x-2)}$$